A new low-and-high gain feedback design using MPC for global stabilization of linear systems subject to input saturation

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Abstract— **In this paper, we develop a new low-and-high gain feedback design methodology using a ultra-short-horizon Model Predictive Controller (MPC) for global asymptotic stabilization of discrete-time linear system subject to input saturation. The proposed method yields improved performance over classical low-and-high gain design and has reduced computational complexity and guaranteed global asymptotic stability of closed-loop system compared to MPC with a long prediction horizon.**

I. Introduction

Stabilization of linear systems subject to actuator saturation has been extensively studied during the past two decades and is still drawing renewed attention, largely because saturation is widely recognized as ubiquitous in engineering applications and inherent constraints in control system designs. Many significant results have already been obtained in the literature. Some early works in this area are summarized in [3], [21], [26], [22], [7], [10] and references therein.

The low-gain method, proposed in [14], [15], [13], was originally developed as a linear feedback design methodology in the context of semi-global stabilization under actuator saturation and later on extended to the global framework with a gain scheduling [18], [6]. The low-gain feedback is parameterized by a so-called low-gain parameter, which is determined a priori in the semi-global setting according to a pre-selected compact set or adaptively with respect to states in the global setting. By properly selecting this parameter, we are able to limit the input magnitude to a sufficiently small level and avoid saturation for all time so as to stabilize the system.

On one hand the low-gain proves to be successful in solving stabilization problems, on the other hand it does not utilize the full actuation level and hence is conservative and less capable regarding performance. Low-and-high gain feedback designs are conceived to rectify the drawbacks of low-gain design methods, and can make better use of available control capacity. As such, they have been used for control problems beyond stabilization, to enhance transient performance and to achieve robust stability and disturbance rejection (see [15], [16], [20], [6], [19], [27]). However, as will be shown in this paper, there is still a room for improvement.

Model predictive controllers (MPC) have a reputation of dealing with constraints and achieving good closed-loop performance. It numerically solves a finite-horizon constrained optimal control problem at each sample. Hence, a MPC may choose to operate exactly at the constraints, while a low-gain strategy would be to avoid the constraints. The more aggressive approach of the MPC complements the more conservative low-gain strategy, and is an interesting approach to include in a low-and-high gain feedback design in order to improve performance.

A drawback of MPC is the computational complexity of solving online numerically a constrained optimization problem (usually a quadratic program) at each sample. Guarantees of MPC stability requires particular formulations of the finite-horizon optimal control problem, such as sufficiently long prediction horizon and the use of a terminal cost, [24], or terminal constraints, [11]. Reduction of computational complexity typically requires that the prediction horizon is made shorter, which comes at the cost of more complex terminal costs and constraints, as well as sub-optimality compared to an infinite horizon constrained optimal control formulation, see e.g. [23], [25] for examples of such reformulations.

Explicit MPC of constrained linear systems admits a piecewise affine state feedback solution to be pre-computed using multi-parametric quadratic programming, [2]. Although online computational complexity can be reduced by orders of magnitude, the approach is still limited by available computer memory and the cost of off-line pre-computations, [29], [1]. Consequently, low-complexity sub-optimal strategies are also of interest in explicit MPC in order to manage complexity due to long prediction horizon, high system order, or many constraints, while preserving stability (see e.g. [9], [12], [5], [8]).

The key idea pursued in this paper is to use an ultra-short-horizon MPC as the high gain strategy in a low-high-gain feedback design, where simple constraints resulting from the low-gain design are imposed on the MPC in order to guarantee stability.

II. Classical low-gain design and MPC

Consider a discrete-time system

$$x_{k+1} = Ax_k + B\sigma(u_k) \tag{1}$$

where $\sigma(\cdot)$ is standard saturation, i.e. for $u \in \mathbb{R}^m$, $\sigma(u) = [\sigma_0(u_1); \cdots ; \sigma_0(u_m)]$, $\sigma_0(u_i) = \operatorname{sign}(u_i)\min\{1, |u_i|\}$ and

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sign(s) is defined as

$$\text{sign}(s) = \begin{cases} 1, & s \geq 0; \\ -1, & s < 0. \end{cases} \quad (2)$$

It is well-known that the global stabilization problem is solvable if and only if the following assumption holds

Assumption 1: (A, B) is stabilizable and A has all its eigenvalues in the closed unit circle.

A. Classical ARE-based low-gain feedback design

The low-gain feedback is a sequence of parameterized feedback gains F_ε satisfying the following properties:

1) $A + BF_\varepsilon$ is Schur stable;
2) $\lim_{\varepsilon\to 0} F_\varepsilon = 0$;
3) $\lim_{\varepsilon\to 0} \|F_\varepsilon(A + BF_\varepsilon)^k x_0\|_{\ell_\infty} = 0$ for any x_0.

The parameter ε is called low-gain parameter. Low-gain feedback can be design using different methods (see [28] and references therein). One way of designing low-gain feedback based on the solution of an H_2 algebraic Riccati equation (ARE) is as follows [17]:

$$u_L = F_\varepsilon x = -(I + B' P_\varepsilon B)^{-1} B' P_\varepsilon A x$$

where P_ε is the positive definite solution of ARE:

$$P_\varepsilon = A' P_\varepsilon A + \varepsilon I - A' P_\varepsilon B(I + B' P_\varepsilon B)^{-1} B' P_\varepsilon A.$$

Following the argument in [17], it is straightforward to show that the control formulation can be generalized by selecting a matrix $R > 0$ and a parameterized matrix $Q_\varepsilon > 0$ such that $\lim_{\varepsilon\to 0} Q_\varepsilon = 0$ and $\frac{\mathrm{d}Q_\varepsilon}{\mathrm{d}\varepsilon} > 0$, and choosing

$$u_L = F_\varepsilon x = -(R + B' P_\varepsilon B)^{-1} B' P_\varepsilon A x \quad (3)$$

where P_ε is the solution of ARE:

$$P_\varepsilon = A' P_\varepsilon A + Q_\varepsilon - A' P_\varepsilon B(R + B' P_\varepsilon B)^{-1} B' P_\varepsilon A, \quad (4)$$

which has the property that $P_\varepsilon \to 0$ as $\varepsilon \to 0$ provided that Assumption 1 holds. It is shown in [17] that (3) satisfies the three low-gain properties.

The low-gain feedback has been successfully employed to solve the semi-global stabilization of a linear system subject to input saturation given by (1). In this context, the low-gain parameter ε controls the domain of attraction of the closed-loop system. It is clear from the properties of low-gain feedback that with a smaller ε, we can shrink the control input to avoid saturation for a large set of initial conditions. Hence by tuning ε sufficiently small, the domain of attraction can be made arbitrarily large to contain any a priori given compact set. To be precise, for any a priori given compact set, say $\mathcal{W}$, there exists ε^* such that for any $\varepsilon \in (0, \varepsilon^*]$, the origin of closed-loop system of (1) and (3) is locally asymptotically stable with $\mathcal{W}$ contained in the domain of attraction. In this case, the resulting low-gain feedback is linear.

In order to solve the global stabilization problem, the low-gain parameter ε can be scheduled adaptively with respect to the states. This has been done in the literature, see for instance [6]. In general, the scheduled parameter $\varepsilon(x)$ should satisfy the following properties:

1) $\varepsilon(x) : \mathbb{R}^n \to (0, 1]$ is continuous and piecewise continuously differentiable.
2) There exists an open neighborhood $\mathcal{O}_s$ of the origin such that $\varepsilon(x) = 1$ for all $x \in \mathcal{O}_s$.
3) For any $x \in \mathbb{R}^n$, we have $\|F_{\varepsilon(x)} x\| \leq 1$.
4) $\varepsilon(x) \to 0$ as $\|x\| \to \infty$.
5) $\{\ x \in \mathbb{R}^n \mid x' P_{\varepsilon(x)} x \leq c\ \}$ is a bounded set for all $c > 0$.

One particular choice of scheduling ε, which satisfies the above conditions, is given in [6] as follows

$$\varepsilon(x) = \max\{r \in (0, 1] \mid (x' P_r x)\,\text{trace}(P_r) \leq \tfrac{1}{b}\} \quad (5)$$

where $b = 2\,\text{trace}(BB')$ while P_r is the unique positive definite solution of ARE (4) with $\varepsilon = r$.

The scheduled version of low-gain feedback controllers for global stabilization is given by

$$u_L(x) = F_{\varepsilon(x)} x = -(B' P_{\varepsilon(x)} B + R)^{-1} B' P_{\varepsilon(x)} A x \quad (6)$$

where $P_{\varepsilon(x)}$ is the solution of (4) with ε replaced by $\varepsilon(x)$.

B. MPC

Let U denote the region $\{u \in \mathbb{R}^m \mid u_i \in [-1, 1], i = 1, ..., m\}$. An MPC problem with prediction horizon N can be formulated by solving the optimization problem

$$\begin{gathered} \min_{\{u_k\}_{k=0}^{N-1}} \sum_{k=0}^{N-1} x_k' Q x_k + u_k' R u_k + x_N' P x_N \\ \text{s.t.} \\ x_{k+1} = Ax_k + Bu_k,\ \forall k = 0, \dots, N-1 \\ u_k \in U,\ \forall k = 0, \dots, N-1, \\ R > 0, \quad Q > 0, \quad P \geq 0 \end{gathered}$$

Under the Assumption 1, the optimization problem is feasible for every initial condition. By solving the above, we obtain an open-loop optimal control sequence $(u_0, \dots, u_{N-1})$. Only the first input is applied to the system. This process is repeated at the each sample time.

The optimization problem can be reformulated in the mp-QP

$$\begin{gathered} J(x_0) = x_0' Y x_0 + \min_{U_N} U_N' H U_N + x_0' F U_N \\ \text{s.t.} \\ G U_N \preceq W + E x_0, \end{gathered}$$

where $H > 0$, $U_N = (u_0', \dots, u_{N-1}')$ is the optimal control sequence and Y, H, F, G, W, E can be obtained from system dynamics and Q, R and P (see [2]).

It is shown in [4] that, by selecting P as the unique positive definite solution of the ARE

$$P = (A - BL')P(A - BL) + L' RL + Q \quad (7)$$

where

$$L = (B' PB + R)^{-1} B' PA,$$

there is a positively invariant region $\mathcal{O}_\infty$ around the equilibrium for which the MPC controller corresponds to $u_k = -Lx_k$, and in which no constraints are activated (i.e., the controller becomes an unconstrained LQR controller).

Moreover, if N is chosen sufficiently large, then the MPC controller is sure to bring any initial condition within $\mathcal{W}$ to $\mathcal{O}_\infty$ within N steps (i.e., by the end of the prediction horizon). In this case, the MPC solution is equivalent to the solution of the infinite-horizon optimization problem

$$\begin{gathered}\min_{\{u_k\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} x_k' Q x_k + u_k' R u_k \\ \text{s.t.} \\ x_{k+1} = Ax_k + Bu_k,\ \forall k = 0, 1, \ldots \\ u_k \in U,\ \forall k = 0, 1, \ldots .\end{gathered}$$

and stability is therefore guaranteed.

C. Connection between scheduled low-gain and MPC

Suppose we choose in scheduled low-gain design that $Q_\varepsilon = \varepsilon Q$ and R to be the same as in MPC. Note that for $x_k \in \mathcal{O}_s$, we have $\varepsilon(x_k) = 1$ and $Q_\varepsilon = Q$. Hence for $x_k \in \mathcal{O}_s \cap \mathcal{O}_\infty$, the scheduled low-gain controller corresponds precisely to the MPC controller $u_k = -Lx_k$. It is also easily verified that the AREs (4) and (7) are the same, with $\varepsilon = 1$.

From the comparison above, we can conclude that the MPC and scheduled low-gain formulations produce an "inner region" $\mathcal{O}_s \cap \mathcal{O}_\infty$ around the equilibrium, where they share an unconstrained optimal linear controller. However, for x outside this region, they determine the control input differently.

In the semi-global case, if we formulate the MPC problem with a weighting matrix Q_ε such that $\lim_{\varepsilon\to 0} Q_\varepsilon = 0$, in the inner region $\mathcal{O}_\infty$, an unconstrained linear controller applies, which corresponds precisely to an ARE-based low-gain controller (3). As $\varepsilon \to 0$, $\mathcal{O}_\infty$ will expand to become arbitrarily large. Eventually, the MPC controller within $\mathcal{W}$ will simply be a linear controller corresponding to a stabilizing ARE-based low-gain controller.

III. Low-and-high gain design using MPC

A. Classical low-and-high-gain feedback design for discrete-time system

Although in the global framework, the low-gain parameter is adapted with respect to the states so that the control input gets as close to the admissible limits as possible while avoiding saturation, the low-gain feedback still does not fully utilize the actuation capability, especially in the MIMO case. To rectify this drawback, the so-called low-and-high gain feedback design method was developed in [6], [19], [27].

The low-and-high-gain state feedback is composed of a low-gain state feedback and a high-gain state feedback as

$$u_k = u_L + u_H = F_{\varepsilon(x_k)} x_k + F_H x_k \tag{8}$$

where $F_{\varepsilon(x_k)} x_k$ is the scheduled low-gain feedback designed in previous section with $R = I$. The high-gain feedback is of the form,

$$F_H x_k = \rho F_{\varepsilon(x_k)} x_k$$

where $\rho > 0$ is called the high-gain parameter.

For continuous-time systems, the high gain parameter ρ does not affect the domain of attraction and can be any positive real number. It aims mainly at achieving control objectives beyond stability, such as robustness, disturbance rejection and performance. However, the high-gain parameter can not be arbitrarily large for discrete-time systems. In order to preserve local asymptotic stability, this high gain has to be bounded at least near the equilibrium. A suitable choice of such a high-gain parameter satisfies

$$\rho \in \left[0, \tfrac{2}{\|B' P_\varepsilon B\|}\right] \tag{9}$$

where P_ε is the solution of ARE (4) with $R = I$ (see [27]). This high-gain can also be adapted respect to states and accompanied with the scheduled low-gain parameter to solve the global stabilization problem. This result is stated in the next lemma [27]:

Lemma 1: Consider system (1). Suppose $R = I$ and $Q_\varepsilon > 0$ is such that

$$\lim_{\varepsilon\to 0} Q_\varepsilon = 0, \quad \tfrac{\mathrm{d}Q_\varepsilon}{\mathrm{d}\varepsilon} > 0 \text{ for } \varepsilon > 0.$$

Let P_ε be the solution of (4). The equilibrium of the interconnection of (1) with the low-and-high-gain feedback

$$u_k = -(1 + \tfrac{2}{\|B' P_{\varepsilon(x_k)} B\|})(I + B' P_{\varepsilon(x_k)} B)^{-1} B' P_{\varepsilon(x_k)} A x_k \tag{10}$$

is globally asymptotically stable.

Proof: For simplicity, we denote $\varepsilon(x_k)$, $F_{\varepsilon(x_k)}$ and $P_{\varepsilon(x_k)}$ respectively by ε_k, F_k and P_k.

Define a Lyapunov function $V_k = x_k' P_k x_k$. The scheduling (5) guarantees that

$$\|(I + B' P_k B)^{-1} B' P_k A x_k\| \le 1.$$

Define $\mu_k = \|B' P_k B\|$, $v_k = -(I + B' P_\varepsilon B)^{-1} B' P_\varepsilon A x_k$ and $\tilde{u}_k = \sigma(u_k)$. We evaluate $V_{k+1} - V_k$ along the trajectories as

$$\begin{aligned} V_{k+1} - V_k &= -x_k' Q_k x_k - \tilde{u}_k' \tilde{u}_k + x_{k+1}' [P_{k+1} - P_k] x_{k+1} \\ &\quad + [\tilde{u}_k - v_k]'(I + B' P_k B)[\tilde{u}_k - v_k] \\ &\le -x_k' Q_k x_k - \|\tilde{u}_k\|^2 + (1 + \mu_k)\|\tilde{u}_k - v_k\|^2 \\ &\quad + x_{k+1}' [P_{k+1} - P_k] x_{k+1} \\ &= -x_k' Q_k x_k + \mu_k \|\tilde{u}_k - \tfrac{1+\mu_k}{\mu_k} v_k\|^2 \\ &\quad - \tfrac{1+\mu_k}{\mu_k}\|v_k\|^2 + x_{k+1}' [P_{k+1} - P_k] x_{k+1}. \end{aligned}$$

Since $\|v_k\| \le 1$, we have

$$\|v_k\| \le \|\tilde{u}_k\| \le (1 + \tfrac{2}{\mu_k})\|v_k\|.$$

This implies that

$$\|\tilde{u}_k - \tfrac{1+\mu_k}{\mu_k} v_k\| \le \tfrac{1}{\mu_k}\|v_k\|,$$

and thus,

$$\mu_k \|\tilde{u}_k - \tfrac{1+\mu_k}{\mu_k} v_k\|^2 - \tfrac{1}{\mu_k}\|v_k\|^2 \le 0.$$

Combining the above, we get for any $x_k \neq 0$,

$$V_{k+1} - V_k \le -x_k' Q_k x_k - \|v_k\|^2 + x_{k+1}' [P_{k+1} - P_k] x_{k+1}.$$

Note that the scheduling (5) implies that $V_{k+1} - V_k$ and $x_{k+1}'[P_{k+1} - P_k] x_{k+1}$ can not have the same signs (see [6]). Consequently, we have for $x_k \neq 0$

$$V_{k+1} - V_k < 0$$

This proves global asymptotic stability of the origin. ■

B. A new low-and-high-gain feedback design using MPC

The underlying philosophy behind the above low-and-high gain design is, based on the low-gain design and its associated Lyapunov function V_k, to find a high gain part and form a composed controller which not only renders V_k to decay every step to ensure stability but also potentially accelerates the convergence.

With this in mind, we shall propose another low-and-high gain design methodology using MPC with a prediction horizon $N = 1$. For $Q > 0$ and $R > 0$, let $Q_\varepsilon = \varepsilon Q$, P_ε be the solution of (4) with Q_ε and R and $\varepsilon = \varepsilon(x_k)$ be determined by (5).

Consider the Lyapunov candidate $V(x_k) = x_k' P_{\varepsilon(x_k)} x_k$. We also take the same abbreviations as used in the proof of Lemma 1. Under the constraints

$$u_k \in U, \ \forall k, \tag{11}$$

we have that for arbitrary u_k along the trajectory of (1),

$$\begin{aligned} V_{k+1} - V_k &= -x_k' Q_k x_k - u_k' R u_k + x_{k+1}' [P_{k+1} - P_k] x_{k+1} \\ &\quad + [u_k - F_k x_k]' (R + B' P_k B)[u_k - F_k x_k] \\ &= -x_k' Q_k x_k - x_k' F_k' R F_k x_k \\ &\quad - 2x_k' F_k' R[u_k - F_k x_k] \\ &\quad + [u_k - F_k x_k]' B' P_k B [u_k - F_k x_k] \\ &\quad + x_{k+1}' [P_{k+1} - P_k] x_{k+1}. \end{aligned}$$

where $F_k = -(R + B' P_k B)^{-1} B' P_k A$. In view of the property that $V_{k+1} - V_k$ can not have the same sign with $x_{k+1}'[P_{k+1} - P_k]x_{k+1}$, to ensure $V_{k+1} - V_k < 0$ for $x_k \neq 0$, it is sufficient to restrict that

$$\begin{aligned} &2x_k' F_k' R[u_k - F_k x_k] \\ &\qquad - [u_k - F_k x_k]' B' P_\varepsilon B [u_k - F_k x_k] \geq 0 \end{aligned} \tag{12}$$

This can be satisfied by enforcing constraints for $i = 1, ...m$,

$$\text{sign}(D_{k,i} x_k)(u_{k,i} - F_{k,i} x_k) \geq 0, \tag{13}$$

$$[2D_{k,i} x_k - C_{k,i}(u_k - F_k x_k)](u_{k,i} - F_{k,i} x_k) \geq 0 \tag{14}$$

where $C_k = B' P_k B$, $D_k = R F_k$ and $C_{k,i}$, $D_{k,i}$, $F_{k,i}$ and $u_{k,i}$ denote the ith row of C_k, D_k, F_k and u_k. Function sign($\cdot$) is defined in (2). Observe that (13) and (14) hold if (13) and the following constraints are satisfied:

$$\text{sign}(D_{k,i} x_k)\left[2D_{k,i} x_k - C_{k,i}(u_k - F_k x_k)\right] \geq 0. \tag{15}$$

Note that (13) and (15) are a conservative reformulation of (12).

The constraints (13) and (15) are linear in u_k only for current step. We can obtain u_k by solving the following MPC problem with $N = 1$

$$\min_{u_k} \ J = u_k' R u_k + x_{k+1}' P x_{k+1} \tag{16}$$

subject to

$$x_{k+1} = A x_k + B u_k \tag{17}$$

$$-1 \leq u_{k,i} \leq 1, \ i = 1, ..., m, \tag{18}$$

$$\text{sign}(D_{k,i} x_k)(u_{k,i} - F_{k,i} x_k) \geq 0, \ i = 1, ..., m, \tag{19}$$

$$\text{sign}(D_{k,i} x_k)\left[2D_{k,i} x_k - C_{k,i}(u_k - F_k x_k)\right] \geq 0, \quad i = 1, ..., m, \tag{20}$$

where P is P_ε with $\varepsilon = 1$.

This problem is always feasible since $u_k = F_k x_k$ is a feasible solution. The solution to the above MPC problem can be obtained by online solving the following convex Quadratic Programming problem

$$\min_{u_k} \ J = u_k'(R + B' P B) u_k + 2x_k' A' P B u_k \tag{21}$$

subject to

$$-1 \leq u_{k,i} \leq 1, \ i = 1, ..., m, \tag{22}$$

$$\text{sign}(D_{k,i} x_k)(u_{k,i} - F_{k,i} x_k) \geq 0, \ i = 1, ..., m, \tag{23}$$

$$\text{sign}(D_{k,i} x_k)\left[2D_{k,i} x_k - C_{k,i}(u_k - F_k x_k)\right] \geq 0, \quad i = 1, ..., m, \tag{24}$$

The resulting u_k is a nonlinear function of x_k, which we can denote as $u_k = f(x_k)$.

Note that by a re-parametrization with introducing more parameters, an explicit solution of (21)-(24) with affine dependence on the parameters can be obtained using multi-parametric quadratic programming (mp-QP). This will increase complexity, but it is expected to contribute less to the added complexity than increasing the prediction horizon N.

We have the following theorem

Theorem 1: The equilibrium of the closed-loop system of (1) and the controller $u_k = f(x_k)$ constructed through (21)-(24) is globally asymptotically stable.

Proof: By construction, the obtained controller u_k guarantees that $V_{k+1} - V_k < 0$ for any $x_k \neq 0$. The result follows immediately. ∎

Remark 1: Compared with the classical low-and-high gain design, the proposed modified approach using MPC yields an LQ optimal controller in a local region around the equilibrium. Moreover, while preserving $V_{k+1} - V_k$, it allows more freedom in choosing u_k when states are large and hence will potentially improve the performance, however at the cost of more computational loads. On the other hand, compared to MPC with a long prediction horizon, the modified approach achieves a guaranteed global asymptotic stability of the closed-loop with very short prediction horizon $N = 1$ and it is more computationally efficient than MPC with large N.

IV. Example and Simulation

Consider the following system

$$x_{k+1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma(u_{k,1}) \\ \sigma(u_{k,2}) \end{bmatrix} \tag{25}$$

Choose $Q = I$ and $R = I$. We simulate the closed-loop systems of (25) with classical low-and-high gain feedback

(10) and modified low-and-high gain feedback defined by (21)-(24). The simulations are initialized from 8 corner points of cubic $[-4, 4]\times[-4, 4]\times[-4, 4]$. The state evolutions are shown in the following figures. On average, we observe a $20\% - 25\%$ improvement on the settling time and overshoot.

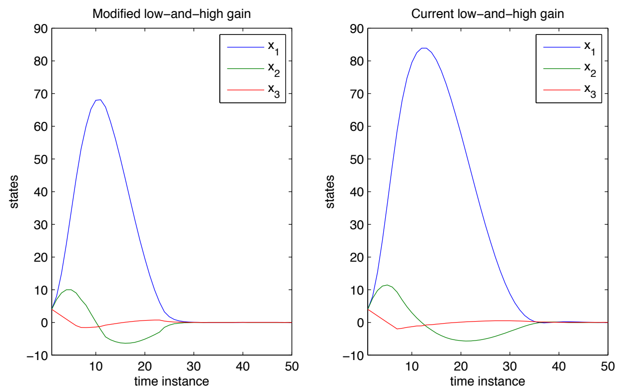

Fig. 1. Initial condition $[4, 4, 4]'$

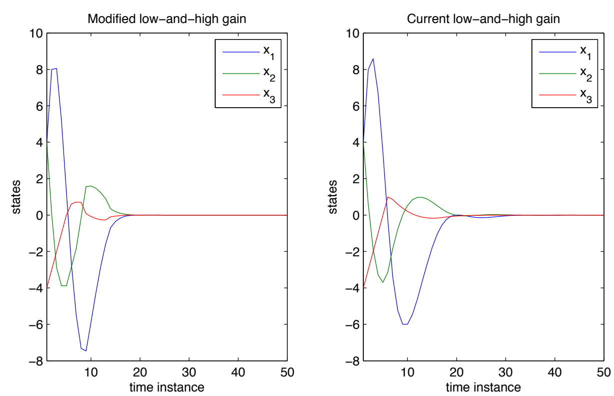

Fig. 2. Initial condition $[4, 4, -4]'$

V. Conclusion

In this paper, we developed a new low-and-high gain feedback design methodology for global stabilization of discrete-time linear systems subject to input saturation using an ultra-short horizon MPC. Simulation on a triple-integrator type system shows improved performance compared with classical low-and-high gain method. The design in this paper can also be done in a semi-global framework, in which scheduling of ε is not needed and computational complexity can be further reduced.

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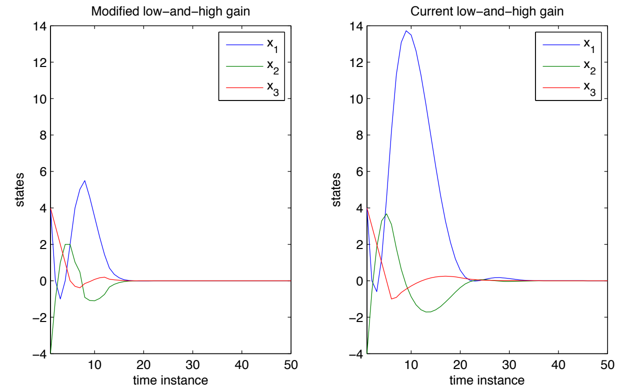

Fig. 3. Initial condition $[4, -4, 4]'$

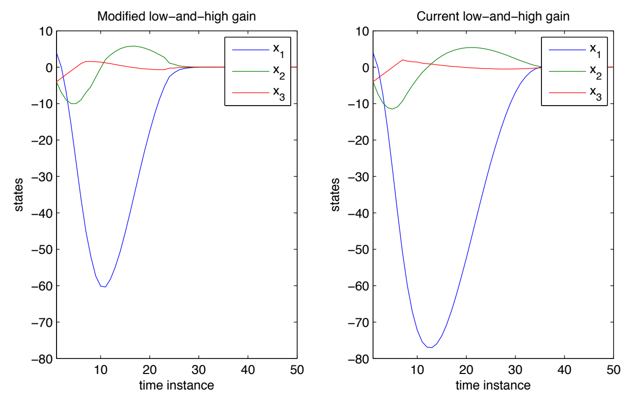

Fig. 4. Initial condition $[4, -4, -4]'$

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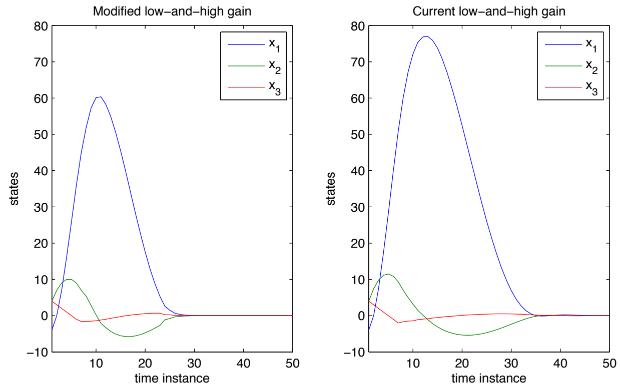

Fig. 5. Initial condition $[-4, 4, 4]'$

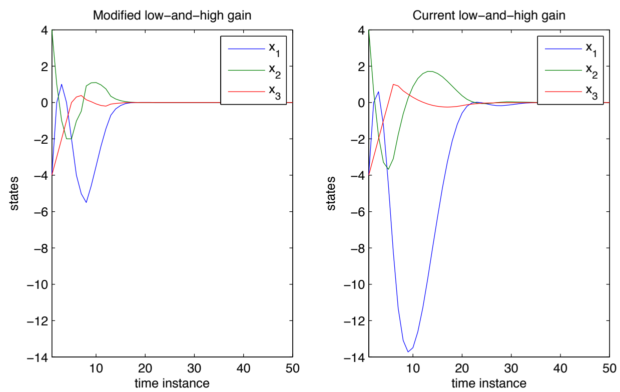

Fig. 6. Initial condition $[-4, 4, -4]'$

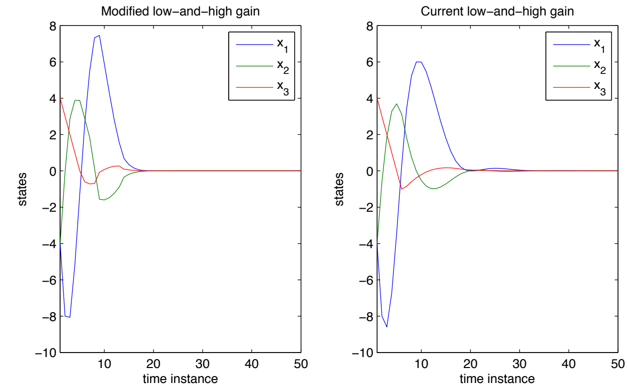

Fig. 7. Initial condition $[-4, -4, 4]'$

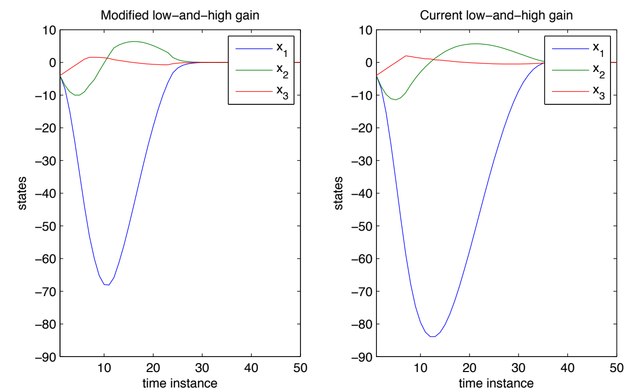

Fig. 8. Initial condition $[-4, -4, -4]'$

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