Autonomous Underwater Vehicle Navigation using Moving Baseline on a Target Ship

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***Abstract*- This paper presents two alternative methods for moving long baseline (MLBL) navigation of autonomous underwater vehicles (AUVs). In both methods 13-bit messages are broadcasted from transponders on the moving target ship to the AUV. These messages provided both range and state information about the target ship. The first method uses an extended Kalman filter (EKF) based on a relative coordinate system fixed to a moving target ship. The second method uses two EKFs, one to estimate the state of the target ship, and a second to estimate the state of the AUV itself. Both methods performed similarly in simulation. Further simulations utilizing field test data showed the global approach to be more robust to the target ship dynamics and message broadcast timing cycle.**

I. INTRODUCTION

The Center for Intelligent Systems Research at the University of Idaho is designing a fleet of Autonomous Underwater Vehicles (AUVs) to measure the magnetic signature of naval vessels. The goal of the magnetic signature mission is to reduce the susceptibility of US naval vessels to magnetically triggered undersea mines. This is accomplished through active magnetic cancelation, which is calibrated with collected magnetic data.

During data collection a fleet of AUVs, equipped with magnetometers, will pass under the target ship while recording the local magnetic field. Each AUV in the fleet must position itself such that all relevant magnetic fields are measured. This scenario requires each AUV to navigate relative to the moving target ship. Long baseline (LBL) systems are used to bound dead-reckoning position estimation and have been shown to meet the accuracy requirements of the magnetic signature mission [1,2]. However, a requirement of traditional LBL systems is the a priori knowledge of the LBL transponder locations. For the magnetic signature mission, it is impractical to deploy and survey an array of temporary LBL transponders. Instead, the LBL transponders will be mounted on the target ship, creating a moving long baseline (MLBL) ranging system. The AUV fleet will range from the MLBL system using synchronous ranging or one-way-travel-time (OWTT) navigation. This will increase the frequency of range measurements by every member of the fleet.

Past work has been conducted on MLBL navigation in [3-14]. In [3-10] MLBL navigation was conducted using the Woods Hole Oceanographic Institute (WHOI) micro-modem 32-byte message, with the message payload containing the position of the transponder. This previous research also used synchronous ranging. In [11] two AUVs used MLBL for cooperative navigation while traveling in a parallel, though diverse, formation. One of the vehicles was equipped with a DVL system; this AUV acted as the MLBL transponder and provided its position and range to the other surveying vehicle. Both lake and sea tests were conducted with promising results. Another MLBL system is presented in [12]. The paper identifies relative and absolute coordinate systems and simulation results are presented for the relative coordinate system. Inter-AUV ranging is used in [13]; the MLBL AUV follows a path that will increase the navigation performance of the surveying AUV. The paper describes the challenges of single beacon navigation. Another approach is presented in [14], where the surveying AUV learns the position of the MLBL AUV. The system uses a relative navigation coordinate system with an extended Kalman filter (EKF) operating in the relative frame. The transponder position estimation is conducted separately from the relative navigation EKF.

In the proposed magnetic sensing mission the WHOI micro-modem 13-bit message was used. The advantage of this message is that the shorter message duration allows for both increased reception rates and more frequent receptions. However, the message has a decreased information bandwidth and the position of the target ship cannot be transmitted at full precision. After a high accuracy initial position fix, the higher rate of range receptions should slow the buildup of position error and decrease the penalty of missed messages.

This paper presents two distinct approaches for incorporating the MLBL range information. Both approaches will use an EKF and will expand upon the work in [1]. The first method adds an EKF to track the position of the target ship. The updates to the target ship EKF are provided via the contents of the 13-bit message payload. The estimates from the target ship EKF are used in the AUV EKF to properly interpret the range information from the moving transponders on the target ship. This model allows the AUV to remain in the global (latitude/longitude) reference frame. Alternatively, the second method operates in the relative reference frame of the target ship. This navigation method must rotate the AUV position with the target ship heading. This is accomplished by a rotation about the origin, with information provided by the 13-bit message payload. This paper presents both simulation and post processed field test data under both scenarios.

II. METHODS

The following sections describe the two proposed methods, global and relative, for AUV navigation using a moving baseline on a target ship. These sections also provide a description of the simulations and post processed field test data used to validate and compare the proposed methods.

A. *Relative Navigation Method*

The relative navigation method fixes the model coordinate frame to the target ship. In this approach, the target ship remains fixed at the origin and the AUV EKF tracks the relative position of the AUV. During navigation the AUV continuously aligns itself with the target ship. The flow of information for the relative navigation method is given in Fig. 1. The target ship sends 13-bit messages containing heading along with synchronous ranges. The AUV EKF rotates its position estimate to align with the updated target ship heading. This estimate is used in the AUV controller to follow a course defined in the relative coordinate system. The AUV EKF updates with AUV speed, heading, and target ship ranges. The model propagates with inertial measurement unit (IMU) angular rates and target ship heading.

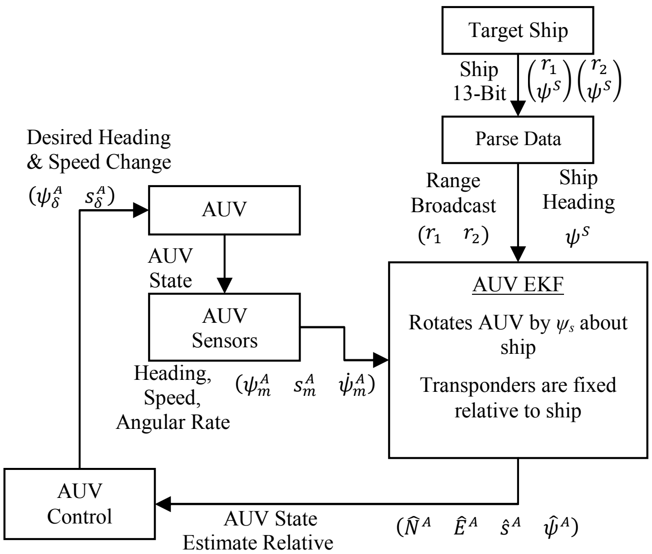

Fig. 1 Flow chart showing the data flow of the relative navigation method. Superscripts *S* and *A* indicate information about the target ship and AUV respectively. A ^ denotes an estimate of the state. A subscript *m* indicates a measurement and a δ indicates a change.

The EKF model expands on the model presented in [1] and accounts for both compass bias and ship induced magnetic compass deflection. Using the EKF notation in [15], the states of the system are

$$X^R = [N^R \quad E^R \quad s^R \quad \psi^R \quad b^R]^T, \tag{1}$$

where X^R is comprised of north, east, speed, heading, and compass bias respectively. The model is expressed as

$$\dot{X}^R = \begin{Bmatrix} \begin{bmatrix} 0 & \dot{\psi}^S \\ -\dot{\psi}^S & 0 \end{bmatrix} \begin{bmatrix} N^R \\ E^R \end{bmatrix} + (\vec{v}^A - \vec{v}^S) \\ 0 \\ \dot{\psi}_m^A \\ 0 \end{Bmatrix}. \tag{2}$$

The model expresses that the rate of change of the position of the ship is equal to the rate of rotation of the ship, $\dot{\psi}^S$, cross the position of the ship plus the relative velocities, *v*, of the AUV and the target ship. The superscripts *S*, *A*, and *R* represent the target ship, the AUV, and the relative frame respectively. The model assumes that the AUV speed and compass bias are constant and the rate of change of heading is equal to the angular rate measured by the IMU, which is included in the EKF model as a driving function.

In order to represent the EKF states in the relative reference frame, the AUV states must be rotated into the reference frame of the target ship. This is accomplished with the rotation matrix

$$R = \begin{bmatrix} \cos(\Delta\psi) & -\sin(\Delta\psi) \\ \sin(\Delta\psi) & \cos(\Delta\psi) \end{bmatrix}, \tag{3}$$

where

$$\Delta\psi = \psi^S - \psi^A. \tag{4}$$

Note that the EKF is estimating the AUV heading, ψ^A. The rotation (3) can be used to rewrite (2). Euler integration produces the discrete propagation model

$$X^R = \begin{Bmatrix} \begin{bmatrix} 0 & \dot{\psi} \\ \dot{\psi} & 0 \end{bmatrix}_{k-1}^S \begin{bmatrix} N \\ E \end{bmatrix}_{k-1}^R \Delta t + (R_{k-1}\vec{v}_{k-1}^A - \vec{v}_{k-1}^S)\Delta t + \begin{bmatrix} N \\ E \end{bmatrix}_{k-1}^R + \begin{bmatrix} w_N \\ w_E \end{bmatrix}^R \\ s_{k-1} + w_s^R \\ \dot{\psi}_{m,k-1}^A \Delta t + \psi_{k-1}^A + w_\psi^R \\ b_{k-1} + w_b^R \end{Bmatrix} \tag{5}$$

where w is the process noise on each state. Linearizing the propagation model about the states yields

$$F_k = \begin{bmatrix} 1 & \dot{\psi}^S \Delta t & \Delta t \cos(\Delta\psi) & \Delta t\, s^R \sin(\Delta\psi) & 0 \\ -\dot{\psi}^S \Delta t & 1 & -\Delta t \sin(\Delta\psi) & \Delta t\, s^R \cos(\Delta\psi) & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{k-1}. \tag{6}$$

All the state noise is considered to be additive such that L_k is the identity matrix.

The measurements available to the EKF are compass heading and forward speed, derived from propeller RPM. Additionally ranges from the target ship are measured when available. During the magnetic signature mission, ranges are available on a 0.5Hz cycle while compass heading and forward speed are measured at 4Hz. The measurement vector can thus be written as

$$y_k = \begin{Bmatrix} \sqrt{(N - B_N)^2 + (E - B_E)^2 + (Z - B_Z)^2} + v_r \\ s + v_s \\ \psi + b + v_\psi \end{Bmatrix}_k, \tag{7}$$

where v is the measurement noise.

Because no absolute position information is needed for the EKF and because speed is approximately constant over the short approach distance, the only piece of information about the target ship needed by the EKF is the heading.

B. *Global Navigation Method*

The global navigation method fixes the model coordinate frame to the earth. In this approach, each AUV uses two separate EKFs to estimate both the state of the target ship and the state and the AUV. The AUV uses this estimate to navigate to a predefined path. The flow of information for the global navigation method is given in Fig. 2.

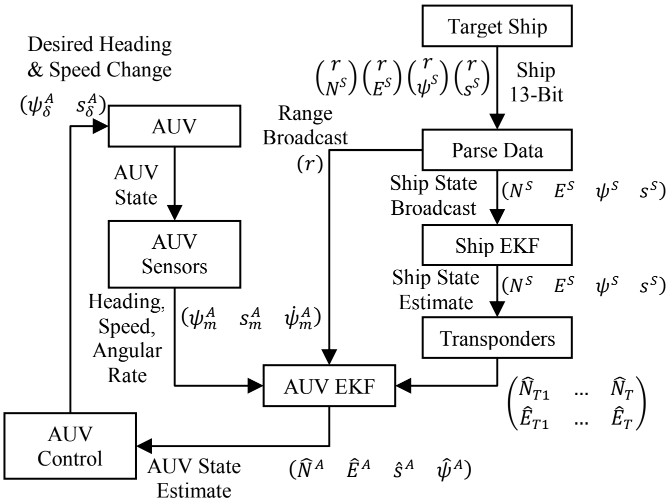

Fig. 2 Flow chart showing the data flow of the global navigation method. Superscripts *S* and *A* indicate information about the target ship and AUV respectively. A ^ denotes an estimate of the state. A subscript *m* indicates a measurement and a δ indicates a change.

Navigation in the fixed frame begins with the target ship broadcast of a 13-bit message. From this message, the AUV receives a synchronous range from one of the transponders and information about the state (north, east, heading, or speed) of the target ship. The received information about the target ship is used in an EKF onboard the AUV to estimate the global position of the target ship, which is then used to estimate the global location of the transponders fixed to the target ship. These estimates, along with the ranges from the 13-bit messages and onboard sensors, are used in a second EKF which estimates the state (north, east, heading, speed, and compass bias) of the AUV itself. The state estimate is used by the AUV controller to navigate along the desired path.

Continuing with the notation used in [15], the system state vector for the EKF to estimate the location of the target ship is

$$X^S = \{N^S \quad E^S \quad s^S \quad \psi^S\}^T, \tag{8}$$

where the system state, X^S, estimates north, east, speed and heading of the target ship, respectively. The superscript *S* designates the target ship. The discrete time system model for the target ship EKF is

$$X_k^S = \left\{\begin{matrix} \Delta t\, s^S \cos\psi^S + N^S + w_{N^S} \\ \Delta t\, s^S \sin\psi^S + E^S + w_{E^S} \\ s^S + w_{s^S} \\ \psi^S + w_{\psi^S} \end{matrix}\right\}_{k-1}, \tag{9}$$

where Δt is the discrete time step. Relating the measurements to the state in the target ship EKF yields

$$y_k^S = \left\{\begin{matrix} N_m^S + v_{N^S} \\ E_m^S + v_{E^S} \\ s_m^S + v_{s^S} \\ \psi_m^S + v_{\psi^S} \end{matrix}\right\}_k, \tag{10}$$

where N_m^S, E_m^S, s_m^S, and ψ_m^S, are the measurements of north, east, speed, and heading of the target ship, respectively. These measurements are extracted from the payloads of the 13-bit messages.

Once the state of the target ship is estimated, the state of the AUV may be estimated. The AUV EKF model uses the model presented in [1] and is given as

$$X^A = \{N^A \quad E^A \quad s^A \quad \psi^A \quad b^A\}^T, \tag{11}$$

where the model estimates north, east, speed, heading, and compass bias of the AUV, respectively. The superscript *A* designates the AUV. The discrete system model for the AUV EKF is

$$X_k^A = \left\{\begin{matrix} \Delta t\, s^A \cos\psi^A + N^A + w_{N^A} \\ \Delta t\, s^A \sin\psi^A + E^A + w_{E^A} \\ s^A + w_{s^A} \\ \Delta t \dot{\psi}_m^A + \psi^A + w_{\psi^A} \\ b^A + w_{b^A} \end{matrix}\right\}_{k-1}, \tag{12}$$

where $\dot{\psi}_m$ is the time rate of change in heading as provided by the IMU and is used as a driving function. The correlation between the measurements and the states for the AUV EKF is

$$y_k^A = \left\{\begin{matrix} \sqrt{(N^A - B_N)^2 + (E^A - B_E)^2 + (Z^A - B_Z)^2} + v_r \\ s^A + v_{s^A} \\ \psi^A + b^A + v_{\psi^A} \end{matrix}\right\}_k \tag{13}$$

There are several advantages to this coordinate frame. An estimate of the global position of both the target ship and each AUV is known throughout the mission, which allows for ease of transition to global navigation if necessary. A potential drawback of this reference frame compared with a relative frame is that it requires a more complex 13-bit message sequence. Another requirement of the fixed frame approach is it requires global positioning information about the target ship.

C. *Simulation*

The performance of the global and relative navigation methods was first evaluated in simulation. The simulations used low-order kinematic models of the motion of AUV and target ship. The models include bounds for both heading rate of change and speed. The AUV and the ship controllers used the MOOS path following algorithm [16] to navigate a waypoint course. For the global navigation method, the waypoints for both the target ship and the AUV were predefined. For the relative navigation method, the AUV waypoints were defined with respect to the target ship coordinate system and therefore move with the ship. Each simulation was conducted with an approach path of 1km with the AUV and target ship heading in opposite directions. The AUV had an initial position offset of 5m in both north and east dimensions. Noise was added to the simulated measurements using magnitudes based on previous field testing results. In addition, a random compass bias was added to the simulated heading data to simulate a poorly calibrated compass. This compass bias was accounted for in the AUV EKF in both methods, as previously described in [1]. The magnitude of the compass bias was set to approximately 1% of path length.

In the simulations two transponders were mounted on the target ship; one on each front corner of the ship. The 13-bit message broadcast timing cycle was based on the planned time interval required for synchronous navigation (for details see

[17]). The message payload cycle depended on which AUV navigation model was used, as seen in Table 1 below:

Table 1. Message broadcast timing cycle used in simulation.

Message Type		Message Cycle Delivery Time (seconds)
Global Method	Relative Method	
North	Heading	0
East	Heading	2
Speed	Heading	4
Heading	Heading	8

In simulation, the quantization of the message payloads was defined as 0.5m for north and east position, 0.024rad for heading, and 0.014m/s for speed.

D. *Simulation using Field Test Data*

The AUVs used in the initial testing of the MLBL navigation methods were part of the University of Idaho mini-AUV fleet described in [2]. These AUVs have similar capabilities to the AUVs that will be used in the magnetic signature mission.

The initial test was designed using two AUVs with one acting as the target ship with a single transponder. The AUV simulating the target ship broadcasted a 13-bit message every 6 seconds, while following a predefined rectangular course. The other AUV followed a larger predefined rectangular course and recorded the 13-bit messages sent by the other vehicle. Both AUVs were navigating independently using the fixed bottom-mounted transponders and conventional LBL.

Each 13-bit message consisted of a 5-bit payload identifier followed by an 8-bit payload. The payload, quantization range and resolution are shown in Table 2.

Table 2. Message payload quantization used in field testing.

Message Payload	Range	Resolution
North	-140m to 0m	0.55m
East	-40m to 80m	0.47m
Speed	0m/s to 2m/s	0.0078m/s
Heading	0rad to 2πrad	0.024rad

Along with the received message, the AUV calculated a synchronous range from the target ship based on the time of travel of the message. The payloads alternate between north, east, speed, and heading of the target ship. For the relative navigation approach, heading was substituted into the payload of each received message in post processing. The timing cycle, described in Table 3 below, was expanded to allow for conventional LBL ranging for each vehicle in-between each 13-bit message broadcast.

Table 3. Message broadcast timing cycle used in field testing.

Message Type		Message Cycle Delivery Time (seconds)
Global Method	Relative Method	
North	Heading	0
East	Heading	6
Speed	Heading	12
Heading	Heading	18

The messages were recorded and used in post processing to evaluate both global and relative navigation methods.

III. RESULTS

The results of the previously described methods both in simulations and field tests are discussed in the sections below.

A. *Simulation*

The previously described simulations provided promising results. During magnetic field reconstruction, relative position is required; because of this, the metric used for evaluation of each navigation method was relative position error. In the relative navigation method, this was calculated by the difference between the actual AUV position and the estimated AUV position. In the global navigation method, the relative position error was calculated as the difference between the magnitudes of the true target ship to true AUV vector and the estimated target ship to estimated AUV vector. This relative error metric was also used to evaluate the results of the field tests presented in the next section.

Both methods were stable and performed similarly with errors typically between 0.5 and 1.0m in the measurement zone. Fig. 3 below shows the results of a typical simulation with the ship operating under high dynamics with a heading oscillation of 3° at a period of 17 seconds. Both methods consistently performed to within the navigation requirement of 1m. However, the relative navigation method had a noticeable approach oscillation. Fig. 4 below shows the oscillation which is primarily due to the target ship frame fixed waypoints oscillating with target ship heading. As the AUV approaches the target ship, the oscillations dampen due to the shorter waypoint distances.

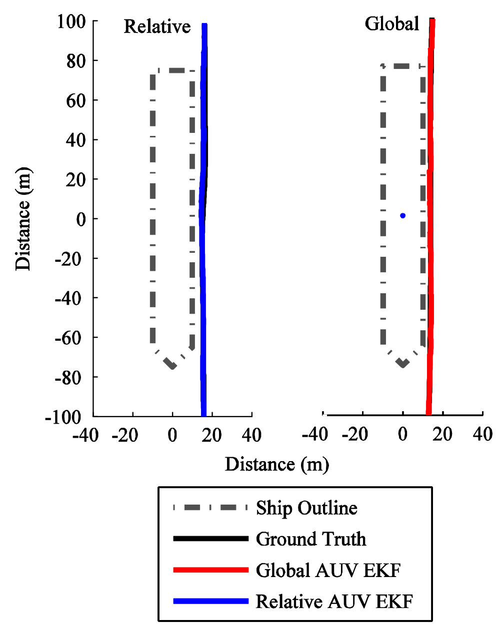

Fig. 3 Simulation Results with high target ship dynamics.

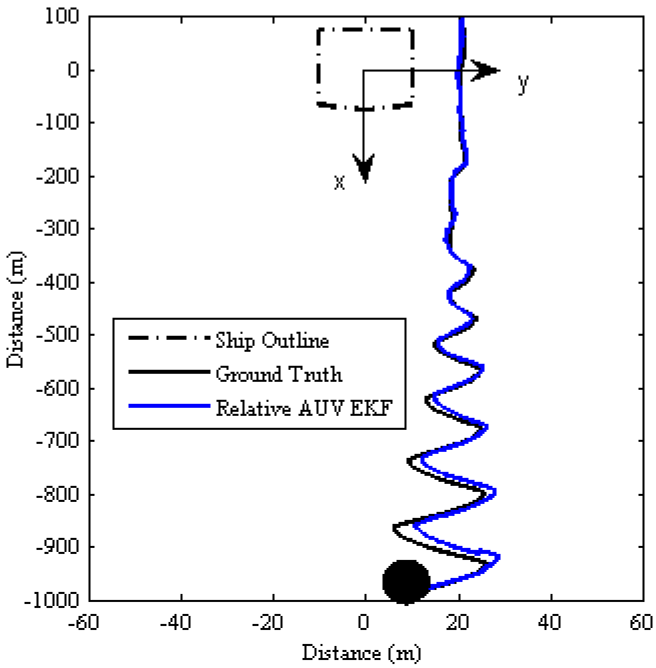

Fig. 4 Relative navigation method simulation showing the AUV approach to the target ship. Oscillation is due to ship frame fixed waypoints oscillating with ship heading. AUV start location is indicated with the black circle. The ship is shown with the dotted outline.

Under expected low ship dynamics the relative navigation method had a mean error of 0.5m with a standard deviation of 1m. The global navigation method had a mean error of 1m with a standard deviation of 0.3m.

B. Simulation using Data from Field Testing

Preliminary field testing provided proof of concept that it is possible to navigate an AUV using only target ship broadcasted messages and ranges. Testing was performed in post processing using both methods with ranges and message payloads collected in field testing.

During testing there was an error in the compass calibration on the AUV acting as the target ship resulting in a compass bias of approximately 10°. This was corrected in the payload of the 13-bit message in post processing. The compass bias also resulted in an impaired controllability of that AUV.

A representative course navigated by the two AUVs is shown in Fig. 5. The AUV start locations were indicated with the circles. The run was divided into two straight passes, but the first pass was heavily affected by the mis-calibrated compass, therefore only one of the passes could be used and it is outlined with a dotted rectangle.

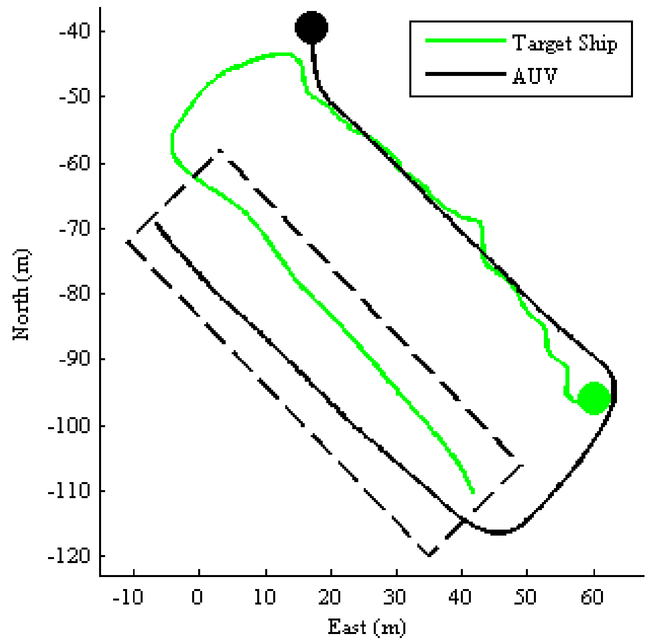

Fig. 5 Field test data collected with two AUVs navigating independently on counter courses. The target ship AUV sent 13-bit messages to the other AUV.

The post processing EKF was utilized as a ground truth benchmark. The post processing EKF has been shown to be accurate to within 70cm [2] and utilized two way ranges from a conventional LBL system.

During the field test the messages broadcast from the target ship AUV to the other AUV were recorded. These messages were used after the field test to simulate navigation using both global and relative navigation methods. The average relative error of the global navigation method was 1.9m using the data from the run shown in Fig. 6 below. The average error between the target ship EKF and the post processed location of the target ship was 1.7 m. This allowed the AUV to navigate to within an average of 0.7m of the post processed position. The short approach length allowed for the improved AUV navigation.

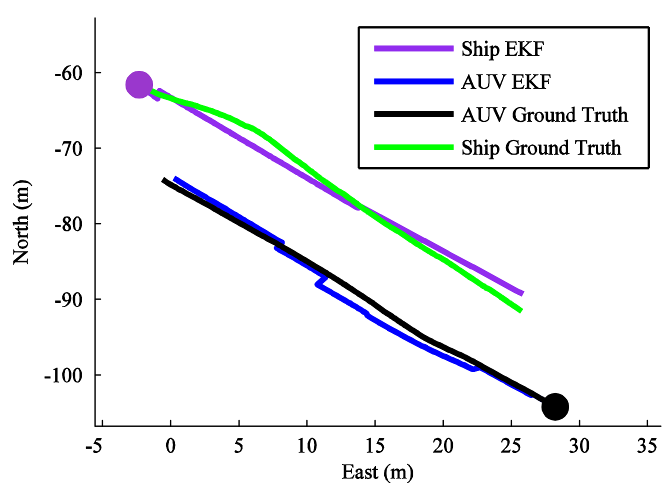

Fig. 6 Comparison of global navigation method to the ground truth showing both AUV and target ship position and estimation. Circles indicate start locations for the target ship and AUV.

Navigation using the relative reference frame yielded a mean error of 2.7m. A comparison of target ship relative positions is shown below in Fig. 7.

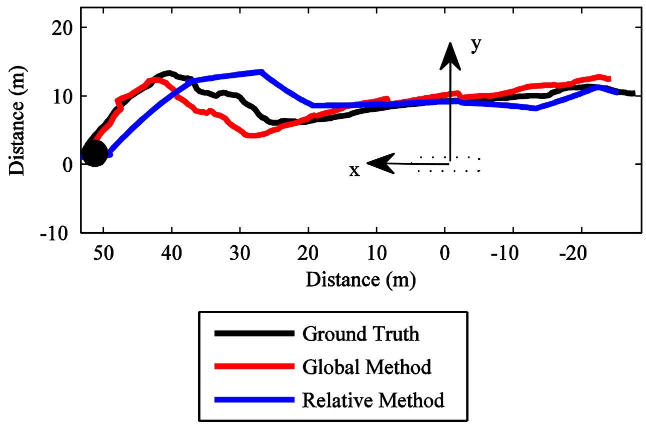

Fig. 7 Comparison of both methods to ground truth in the target ship relative coordinate system. Target ship shown as dotted outline with axis noted. AUV start location identified by black circle.

The relative navigation method is significantly lagging the ground truth. However, it is parallel to the ground truth and with more frequent target ship heading updates the delay could converge to the ground truth path. The delay is created by the discrete derivative of the heading of the target ship which is made more prominent by the lengthened timing cycle as compared with simulation. When the target ship heading stabilized the estimate improved significantly.

The global navigation method performed more accurately. It is apparent that the range updates and message payloads improved the navigation over dead reckoning. Also, the method was robust to the target ship heading changes and did not exhibit the lagging behavior seen in the relative navigation method.

Fig. 8 below shows the position error of both methods when compared to the ground truth.

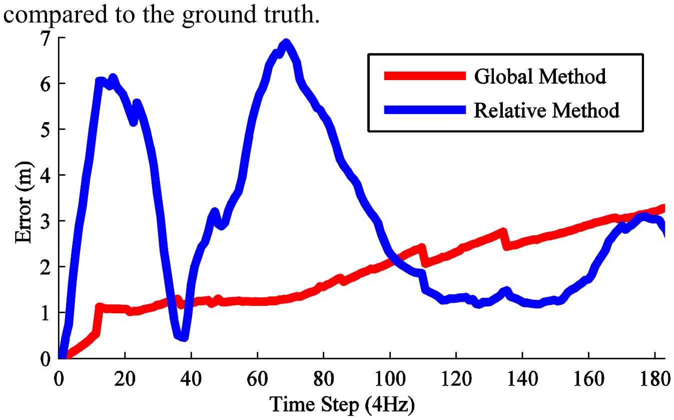

Fig. 8 Comparison of the errors in the methods when compared to the ground truth in target ship coordinate system.

CONCLUSION

Two MLBL navigation methods were designed for the magnetic signature mission. Both methods utilized the WHOI micro-modem 13-bit message to send information from the target ship to the AUV.

The global navigation method received north, east, speed, and heading of the target ship. This information was used in a target ship EKF, onboard the AUV, which then supplied MLBL transponder locations to the AUV EKF for range updates. The relative navigation method mapped the position of the AUV to the target ship coordinate system. This mapping requires the AUV position to rotate with the target ship. However the location of the ship and its transponders remain constant. Because no global information is needed only target ship heading is sent to the AUV.

Simulations were conducted using the magnetic signature mission course. Realistic sensor noise and range error as well as 13-bit message payload quantization was included in the simulations. The simulations used both navigation methods under a variety of target ship dynamics. Both methods performed within the magnetic signature mission requirements.

Initial field tests were conducted to determine the feasibility of MLBL navigation. Data were collected using two independently navigating vehicles with one vehicle acting as the target ship. Acoustic messages were sent at a predefined timing cycle from the target ship AUV to the other AUV with the appropriate message payload and synchronous range. MLBL navigation was then simulated in post processing.

The relative navigation method had trouble with the less frequent broadcasts and varying target ship dynamics. However, when the target ship dynamics stabilized the relative error was reduced, and both methods performed similarly. The global navigation proved to be more robust and performed more accurately over the length of the run. The error in the zone where the two vehicles paths crossed was approximately 2m.

Future work will incorporate an increase in the length of the approach and the number of target ship transponders. The target ship will be replaced with a surface vehicle with multiple transponders. In addition, the methods will be applied in real time and used for active navigation.

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